ROUND z-FILTERS AND ROUND SUBSETS OF βX

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ABSTRACT

A characterization of remote points in $\beta\mathbf{R}$ is given by means of prime z-filters and is generalized to z-filters on arbitrary completely regular Hausdorff spaces X and to subsets of βX.

1. **Introduction.** An interesting topological feature of the Stone-Čech compactification $\beta\mathbf{R}$ of the real line is the existence of a *remote point* in $\beta\mathbf{R}$, i.e., a point not in the closure of any discrete subset of $\mathbf{R}$. Other characterizations and properties of remote points are given in [1], [6], [7] and [8]. The existence of remote points in $\beta\mathbf{R}$ has been demonstrated, assuming the continuum hypothesis, in [1] and in [8]. The present paper was motivated by the following characterization of remote points.

THEOREM. *A prime z-filter $\mathscr{P}$ on $\mathbf{R}$ has the property that for every $Z \in \mathscr{P}$ there exists $W \in \mathscr{P}$ such that $W \subseteq \operatorname{int} Z$ if and only if $\mathscr{P} = \mathscr{M}^p$ for some remote point p in $\beta\mathbf{R}$.*

The proof will be given in Section 2. For terminology and notation see [2]; we use $\mathscr{M}^p$ and $\mathscr{O}^p$ to denote the z-filters $Z[M^p]$ and $Z[O^p]$. We generalize the property of the theorem to an arbitrary completely regular Hausdorff space X as follows.

DEFINITION. A z-filter $\mathscr{F}$ on X will be called *round* if for every $Z \in \mathscr{F}$ there is $W \in \mathscr{F}$ and a cozero-set S in X such that $W \subseteq S \subseteq Z$.

The term "round" is borrowed from [5]. It is used in [10] in connection with proximity spaces. In fact, round z-filters are just the intersections with $Z(X)$ of the round filters on X with respect to the proximity induced by βX. In the case of the real line, every open set is a cozero-set, so our condition reduces to that of the theorem above.

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The basic properties of round z-filters will be obtained in Section 3. *Round subsets of* βX are introduced in Section 4; they are related to an interesting class of round z-filters. In Section 5 we find that every G_δ in βX that does not meet X is a round subset of βX, and we relate this result to the family of continuous functions with compact support.

2. **Round prime z-filters on the real line.** In the proof of the theorem stated in the introduction we shall need some results from [6] which we restate in the first two lemmas below.

LEMMA 2.1. *A prime z-filter on* $\mathbf{R}$ *is minimal if and only if each of its members has nonempty interior.* [6, Theorem 8.1].

LEMMA 2.2. *A point p in* $\beta\mathbf{R}$ *is a remote point if and only if the z-ultrafilter* $\mathcal{M}^p$ *is a minimal prime z-filter.* (Part of [6, Theorem 11.2].)

Lemma 2.1 shows that a nonminimal prime z-filter on $\mathbf{R}$ has a nowhere dense member. We shall also need the following stronger result. We use the symbol $\subset$ to denote proper inclusion.

LEMMA 2.3. *If* $\mathcal{P}$ *and* $\mathcal{Q}$ *are prime z-filters on* $\mathbf{R}$ *with* $\mathcal{P} \subset \mathcal{Q}$, *then there exists* $Z \in \mathcal{P}$ *such that* bdry $Z \in \mathcal{Q}$.

Proof. Since $\mathcal{Q}$ is nonminimal, we may choose a nowhere dense $F \in \mathcal{Q}$. By enlarging F slightly if necessary (e.g., by adjoining the integers), we may assume that all the components of $\mathbf{R} - F$ are bounded. Let U be the union of all the left open thirds of these components, and V of the right. Since cl U contains the set of left endpoints, which is dense in F, we have $F \subseteq \mathrm{cl}U$, and similarly for V. Let Z and W be the complements of U and V in $\mathbf{R}$. Since U and V are disjoint, we have $Z \cup W = \mathbf{R}$. Say $Z \in \mathcal{P}$. Since $F \subseteq \mathrm{cl}(\mathbf{R} - Z)$, we have $F \cap Z \subseteq \mathrm{bdry}\, Z$ and hence bdry $Z \in \mathcal{Q}$.

The proof also shows that on the real line every nowhere dense set is regularly nowhere dense in the sense of [3]. (The author wishes to thank Jack R. Porter for this remark and also for a simplification included in the above proof.)

THEOREM 2.4. *A prime z-filter* $\mathcal{P}$ *on* $\mathbf{R}$ *is round if and only if* $\mathcal{P} = \mathcal{M}^p$ *for some remote point p in* $\beta\mathbf{R}$.

Proof. Let $\mathcal{P}$ be round. Suppose $\mathcal{P} \subset \mathcal{Q}$ for some (prime) z-filter $\mathcal{Q}$. Choose $Z \in \mathcal{P}$ such that bdry $Z \in \mathcal{Q}$ and choose $W \in \mathcal{P}$ such that $W \subseteq \mathrm{int}\, Z$. Then $\mathcal{Q}$ contains the disjoint sets W and bdry Z, which is absurd. Hence $\mathscr{P}$ is a z-ultrafilter and $\mathscr{P} = \mathscr{M}^p$ for some $p \in \beta\mathbf{R}$. Since $\mathscr{P}$ is clearly minimal, p is a remote point.

Conversely, let $\mathscr{P} = \mathscr{M}^p$ for a remote point p. Let $Z \in \mathscr{P}$. Since $\mathscr{P}$ is minimal, bdry $Z \notin \mathscr{P}$. Thus, since $\mathscr{P}$ is a z-ultrafilter, there is $F \in \mathscr{P}$ with $F \cap$ bdry $Z = \varnothing$. Hence $W = F \cap Z \subseteq \operatorname{int} Z$.

3. **Round z-filters.** In this section we develop the basic properties of round z-filters. An auxiliary result is the determination in Theorem 3.5 of the z-filters on X with a prescribed set of cluster points in βX.

DEFINITION. For any z-filter $\mathscr{F}$ on X, we denote by $\mathscr{F}^0$ the family of all zero-sets Z in X such that there is a member W of $\mathscr{F}$ and a cozero-set S in X such that $W \subseteq S \subseteq Z$.

LEMMA 3.1. *For any z-filter $\mathscr{F}$, the family $\mathscr{F}^0$ is a round z-filter; $\mathscr{F}$ is round if and only if $\mathscr{F} = \mathscr{F}^0$.*

Proof. It is easy to verify that $\mathscr{F}^0$ is a z-filter. Let $Z \in \mathscr{F}^0$ and choose $W \in \mathscr{F}$ and a cozero-set S such that $W \subseteq S \subseteq Z$. Since W and $X - S$ are disjoint zero-sets, they are completely separated. Hence by [2, 1.15(a)] there is a zero-set F and a cozero-set T such that $W \subseteq T \subseteq F \subseteq S \subseteq Z$. Hence $F \in \mathscr{F}^0$; this shows that $\mathscr{F}^0$ is round. The last statement is clear.

We adapt the notation of [2, 7O] to z-filters as follows.

DEFINITION. For any z-filter $\mathscr{F}$ on X, we denote by $\theta(\mathscr{F})$ the set of all cluster points of $\mathscr{F}$ in βX. That is,

$$\theta(\mathscr{F}) = \cap_{Z \in \mathscr{F}} \mathrm{cl}_{\beta X} Z = \{p \in \beta X : \mathscr{F} \subseteq \mathscr{M}^p\}.$$

Every nonempty closed subset A of βX is of the form $\theta(\mathscr{F})$; for example, we may take $\mathscr{F} = \{Z \in \mathbf{Z}(X) : A \subseteq \mathrm{cl}_{\beta X} Z\} = \cap_{p \in A} \mathscr{M}^p$.

The following useful result arises in the proof of [2, 7O.2].

LEMMA 3.2. *If $\mathscr{F}$ is any z-filter on X and Z is a zero-set in X such that $\mathrm{cl}_{\beta X} Z$ is a neighborhood of $\theta(\mathscr{F})$, then there exists $W \in \mathscr{F}$ such that $\mathrm{cl}_{\beta X} Z$ is a neighborhood of $\mathrm{cl}_{\beta X} W$.*

Proof. The family of all sets of the form $\mathrm{cl}_{\beta X} W$, where $W \in \mathscr{F}$, is closed under finite intersection and has intersection $\theta(\mathscr{F})$. Since $\theta(\mathscr{F}) \cap (\beta X - \mathrm{int}_{\beta X} \mathrm{cl}_{\beta X} Z) = \varnothing$, by compactness there is $W \in \mathscr{F}$ such that $\mathrm{cl}_{\beta X} W \cap (\beta X - \mathrm{int}_{\beta X} \mathrm{cl}_{\beta X} Z) = \varnothing$.

THEOREM 3.3. *If $\mathscr{F}$ is any z-filter on X, the following are equivalent.*

(a) *$\mathscr{F}$ is a round z-filter.*

(b) *For every* $Z \in \mathscr{F}$ *there is* $W \in \mathscr{F}$ *such that* $\mathrm{cl}_{\beta X}Z$ *is a neighborhood of* $\mathrm{cl}_{\beta X}W$.

(c) *For any* $p \in \beta X$, $\mathscr{F} \subseteq \mathscr{M}^p$ *implies* $\mathscr{F} \subseteq \mathscr{O}^p$.

(d) *For every* $Z \in \mathscr{F}$, $\mathrm{cl}_{\beta X}Z$ *is a neighborhood of* $\theta(\mathscr{F})$.

Proof. (a) is equivalent to (b) by [2, 7.14].

(b) implies (c). If $Z \in \mathscr{F}$ and W is chosen as in (b), then $W \in \mathscr{M}^p$; hence $p \in \mathrm{cl}_{\beta X}W$, $\mathrm{cl}_{\beta X}Z$ is a neighborhood of p, and $Z \in \mathscr{O}^p$.

(c) implies (d). If $p \in \theta(\mathscr{F})$, then $\mathscr{F} \subseteq \mathscr{M}^p$; hence $\mathscr{F} \subseteq \mathscr{O}^p$ and $\mathrm{cl}_{\beta X}Z$ is a neighborhood of p.

(d) implies (b). Lemma 3.2.

We now determine which z-filters on X have a prescribed set of cluster points in βX. The lemma below extends [2, 7H.1]. The theorem generalizes [2, 7.13]; the necessity is [2, 7O.2].

LEMMA 3.4. *Any closed subset A of βX has a base of neighborhoods of the form* $\mathrm{cl}_{\beta X}Z$ *with* $Z \in \boldsymbol{Z}(X)$.

Proof. If U is any open neighborhood of A in βX, then A and $\beta X - U$ are completely separated. Hence there is a zero-set-neighborhood W of A in βX with $W \subseteq U$. Put $Z = W \cap X$. Then $Z \in \boldsymbol{Z}(X)$ and $\mathrm{cl}_{\beta X}Z$ is clearly a neighborhood of A that is contained in U.

THEOREM 3.5. *Let A be any closed subset of βX. For any z-filter $\mathscr{F}$ on X, we have* $\theta(\mathscr{F}) = A$ *if and only if* $\bigcap_{p \in A}\mathscr{O}^p \subseteq \mathscr{F} \subseteq \bigcap_{p \in A}\mathscr{M}^p$.

Proof. Let $\theta(\mathscr{F}) = A$. If $Z \in \bigcap_{p \in A}\mathscr{O}^p$ then $\mathrm{cl}_{\beta X}Z$ is a neighborhood of $\theta(\mathscr{F})$. Thus by Lemma 3.2 there is $W \in \mathscr{F}$ such that $\mathrm{cl}_{\beta X}Z$ contains $\mathrm{cl}_{\beta X}W$; hence $W \subseteq Z$ and so $Z \in \mathscr{F}$. Conversely, we clearly have $A \subseteq \theta(\mathscr{F})$. By Lemma 3.4, $\theta(\bigcap_{p \in A}\mathscr{O}^p) = A$ and hence $\theta(\mathscr{F}) \subseteq A$.

By means of the next two results we completely characterize round z-filters in terms of intersections of the z-filters $\mathscr{O}^p$.

THEOREM 3.6. *For any z-filter $\mathscr{F}$ on X, we have* $\mathscr{F}^0 = \bigcap_{p \in \theta(\mathscr{F})}\mathscr{O}_p$.

Proof. $Z \in \mathscr{F}^0$ if and only if there is $W \in \mathscr{F}$ such that $W \subseteq S \subseteq Z$ for some cozero-set S in X; equivalently by [2, 7.14], such that $\mathrm{cl}_{\beta X}Z$ is a neighborhood of $\mathrm{cl}_{\beta X}W$. Also, $Z \in \bigcap_{p \in \theta(\mathscr{F})}\mathscr{O}^p$ if and only if $\mathrm{cl}_{\beta X}Z$ is a neighborhood of $\theta(\mathscr{F})$; equivalently by Lemma 3.2, a neighborhood of $\mathrm{cl}_{\beta X}W$ for some $W \in \mathscr{F}$.

THEOREM 3.7. *If* $\mathscr{F}$ *is a round z-filter on X, then* $\mathscr{F} = \bigcap_{p \in \theta(\mathscr{F})} \mathscr{O}^p$. *Conversely, if A is any nonempty closed subset of* βX, *then* $\bigcap_{p \in A} \mathscr{O}^p$ *is a round z-filter; these are distinct for distinct closed sets A.*

Proof. The first statement follows from Lemma 3.1 and the last theorem. Now let A be a nonempty closed subset of βX and put $\mathscr{F} = \bigcap_{p \in A} \mathscr{O}^p$. By Theorem 3.5, $\theta(\mathscr{F}) = A$ and hence $\mathscr{F}$ is round by Theorem 3.3(d). The last statement also follows from Theorem 3.5.

COROLLARY 3.8. $A \to \bigcap_{p \in A} \mathscr{O}^p$ *is a one-one order-reversing correspondence between the nonempty closed subsets of* βX *and the round z-filters on X.*

COROLLARY 3.9. *A round z-filter is a z-filter that is minimal with respect to its set of cluster points.*

If $\mathscr{P}$ is a prime z-filter, then $\theta(\mathscr{P})$ has just one point; hence every round prime filter is $\mathscr{O}^p$ for some $p \in \beta X$. Since on the real line $\mathscr{O}^p$ is prime if and only if p is a remote point in $\beta \mathbf{R}$ [6, Theorem 11.2], this provides another proof of Theorem 2.4. Alternatively, Theorem 3.3(c) could be used. However, Lemma 2.3 and the direct proof given in Section 2 are of independent interest and do not require the stronger result from [6] that if $\mathscr{O}^p$ is prime then p is a remote point. In fact, Section 2 and Theorem 3.7 provide another proof of this result.

4. **Round subsets of** βX. Lemma 2.2 shows that p is a remote point in $\beta \mathbf{R}$ *if and only if* $\mathscr{M}^p = \mathscr{O}^p$, i.e., for any $Z \in \mathbf{Z}(\mathbf{R})$, if $\mathrm{cl}_{\beta \mathbf{R}} Z$ contains p, then it is a neighborhood of p. In Section 2 we found the relation between remote points and round prime z-filters. We now generalize the above characterization of remote points, obtaining a class of subsets of βX which is related to a larger class of round z-filters.

DEFINITION. A subset A of βX will be called a *round subset of* βX if for any $Z \in \mathbf{Z}(X)$, if $\mathrm{cl}_{\beta X} Z$ contains A, then it is a neighborhood of A.

We collect in the next theorem some immediate properties of round subsets.

THEOREM 4.1. *Let* $A \subseteq \beta X$.

(a) *A is a round subset of* βX *if and only if* $\bigcap_{p \in A} \mathscr{M}^p = \bigcap_{p \in A} \mathscr{O}^p$.

(b) *If* $cl_{\beta X} A$ *is a round subset of* βX, *then A is also round.*

(c) *Every open subset of* βX *is round.*

(d) *Any union of round subsets of* βX *is also round.*

THEOREM 4.2. *For any nonempty closed subset A of βX, the following are equivalent.*

(a) *A is a round subset of βX.*

(b) *$\bigcap_{p\in A}\mathcal{M}^p$ is a round z-filter.*

(c) *$\bigcap_{p\in A}\mathcal{M}^p = \bigcap_{p\in A}\mathcal{O}^p$.*

(d) *There is a unique z-filter $\mathcal{F}$ on X such that $\theta(\mathcal{F}) = A$.*

Proof. (a) implies (b). We apply Theorem 3.3(d). Since A is closed, $\theta(\bigcap_{p\in A}\mathcal{M}^p) = A$. For every $Z \in \bigcap_{p\in A}\mathcal{M}^p$, $\mathrm{cl}_{\beta X}Z$ contains A, and is hence a neighborhood of A. Thus $\bigcap_{p\in A}\mathcal{M}^p$ is a round z-filter.

(b) implies (c). Since $\theta(\bigcap_{p\in A}\mathcal{M}^p) = A$, this follows from Theorem 3.7.

(c) implies (a). Theorem 4.1(a).

(c) and (d) are equivalent by Theorem 3.5.

EXAMPLE 4.3. Let X be connected and let Z be any proper zero-set in X with nonempty interior. Let A be the interior of $\mathrm{cl}_{\beta X}Z$ and let B be the closure of A. Since A is open, it is a round subset of βX. Since βX is connected, A is not closed. Hence $\mathrm{cl}_{\beta X}Z$ contains B but is not a neighborhood of B. Thus the closure of a round subset of βX need not be round; i.e., the converse of Theorem 4.1(b) is not true. We also have $\bigcap_{p\in A}\mathcal{O}^p \neq \bigcap_{p\in B}\mathcal{O}^p$, whereas the corresponding z-filters using $\mathcal{M}^p$ are equal for any $A \subseteq \beta X$ and any X. Furthermore, $\bigcap_{p\in A}\mathcal{M}^p$ is not round, since it is equal to $\bigcap_{p\in B}\mathcal{M}^p$ which is not round by Theorem 4.2. Thus the hypothesis that A is closed in Theorem 4.2 ((a) implies (b)) may not be removed. Since A is round, we have $\bigcap_{p\in A}\mathcal{M}^p = \bigcap_{p\in A}\mathcal{O}^p$ and hence we see that $\bigcap_{p\in A}\mathcal{O}^p$ is not round. Thus the hypothesis that A is closed in the second part of Theorem 3.7 may not be removed, even if A is round. Also, the intersection of a family of round z-filters need not be round.

5. **Functions with compact support.** Theorem 8.19 of [2] (see also [9]) shows that when X is realcompact, the intersection of all the free maximal ideals in $C(X)$ is the family $C_K(X)$ of all functions with compact support. This property of a realcompact space may be restated in terms of round subsets of βX. For any space X, [2, 7E] shows that $C_K(X) = \bigcap_{p\in\beta X - X}O^p$. Thus the following is immediate.

THEOREM 5.1. *The intersection of all the free maximal ideals in $C(X)$ is the family $C_K(X)$ of all functions with compact support if and only if $\beta X - X$ is a round subset of βX.*

Thus [2, 8.19] says that if X is realcompact, then $\beta X - X$ is a round subset of βX. This will be generalized in Theorem 5.3 below. We first state a well-known lemma.

LEMMA 5.2. *The following are equivalent.*

(a) *X is realcompact.*

(b) *$\beta X - X$ is a union of zero-sets in βX.*

(c) *$\beta X - X$ is a union of G_δ-sets in βX.*

Proof. (a) implies (b). If $p \in \beta X - X$ then M^{*p} contains a unit of C [2, 7.9(b)], i.e., there is $f \in C^*(X)$ such that $p \in Z(f^\beta)$ and $Z(f^\beta) \subseteq \beta X - X$.

(b) implies (c). Trivial.

(c) implies (a). The complement of a G_δ is an F_σ, hence (in the compact space βX) σ-compact and thus realcompact. Hence X is an intersection of realcompact subspaces of βX, and thus by Theorem 8.9 of [2] is itself realcompact.

THEOREM 5.3. *For any space X, every G_δ in βX that does not meet X is a round subset of βX.*

Proof. It follows from the lemma that if a subset of βX does not meet X and is a union of G_δ-sets, then it is a union of zero-sets. Thus, since the family of round subsets of βX is closed under arbitrary unions, it suffices to prove the theorem in the case of a zero-set W in βX that does not meet X. Let $W \subseteq \mathrm{cl}_{\beta X} Z$ for some $Z \in Z(X)$. Put $T = \beta X - W$, choose $f \in C(\beta X)$ so that $W = Z(f)$, and put $h = 1/f$ on T. Suppose W meets $\mathrm{cl}_{\beta X}(X - Z)$. Then h, which is continuous on T is unbounded on $X - Z$; thus $X - Z$ contains a noncompact set S that is C-embedded and closed in T [2, 1.20]. Thus S is C-embedded in X and is thus completely separated in X from the zero-set Z [2, 1.18]; hence S and Z have disjoint closures in βX. Since S is closed in T but not compact we may choose $p \in \mathrm{cl}_{\beta X} S - T$. Thus $p \in W$ but $p \notin \mathrm{cl}_{\beta X} Z$, contradicting our assumption concerning Z. It follows that $W \cap \mathrm{cl}_{\beta X}(X - Z) = \emptyset$. Thus $W \subseteq \beta X - \mathrm{cl}_{\beta X}(X - Z) \subseteq \mathrm{cl}_{\beta X} Z$ and $\mathrm{cl}_{\beta X} Z$ is a neighborhood of W. Hence W is a round subset of βX.

COROLLARY 5.4. *For any space X, $\beta X - \upsilon X$ is a round subset of βX.*

Proof. Put $Y = \upsilon X$. Then Y is realcompact, so $\beta Y - Y$ is a union of G_δ-sets in $\beta Y = \beta X$. Since these G_δ-sets do not meet X, they are round subsets of βX, and hence their union $\beta X - \upsilon X$ is also a round subset of βX.

COROLLARY 5.5.[2, Theorem 8.19]. *If X is realcompact, then $\beta X - X$ is a* *round subset of βX; i.e., the intersection of all the free maximal ideals in $C(X)$ is the family $C_K(X)$ of all functions with compact support.*

It is shown in [4, 3.9] that if X is a P-space, then the intersection of the free maximal ideals in $C(X)$ is the family of functions with compact support. This result may be extended as follows.

THEOREM 5.6. *X is a P-space if and only if every subset of βX is round.*

Proof. X is a P-space if and only if $\mathcal{M}^p = \mathcal{O}^p$ for all $p \in \beta X$ [2, 7L.1]; the result thus follows from Theorem 4.1(a).

EXAMPLE 5.7. For any $p \in \beta\mathbf{R} - \mathbf{R}$, it is clear that $\{p\}$ is a round subset of $\beta\mathbf{R}$ if and only if p is a remote point in $\beta\mathbf{R}$. Under the continuum hypothesis, the set Γ of remote points in $\beta\mathbf{R}$ is dense in $\beta\mathbf{R} - \mathbf{R}$ ([1, 2.5] or [8, 5.4]). Hence every (round) open subset of $\beta\mathbf{R}$ that meets $\beta\mathbf{R} - \mathbf{R}$ contains a remote point, However, not every round subset of $\beta\mathbf{R}$ that meets $\beta\mathbf{R} - \mathbf{R}$ contains a remote point. For example, the set Δ of points in $\beta\mathbf{R} - \mathbf{R}$ that are not remote points in $\beta\mathbf{R}$ is also dense in $\beta\mathbf{R} - \mathbf{R}$ [1, 3.3]. By Corollary 5.5, $\mathrm{cl}_{\beta\mathbf{R}}\Delta = \beta\mathbf{R} - \mathbf{R}$ is a round subset of $\beta\mathbf{R}$. Hence Δ is a round subset of $\beta\mathbf{R}$ but Δ contains no remote point.

REFERENCES

1. N. J. Fine and L. Gillman, *Remote points in* βR, Proc. Amer. Math. Soc. **13** (1962), 29–36.

2. L. Gillman and M. Jerison, *Rings of continuous functions*, Van Nostrand, Princeton, N.J., 1960.

3. M. Katětov, *On the equivalence of certain types of extension of topological spaces*, Časopis Pěst. Mat. Fys. **72** (1947), 101–106.

4. C. W. Kohls, *Ideals in rings of continuous functions*, Fund. Math. **45** (1958), 28–50.

5. H. J. Kowalsky, *Topologische Räume*, Birkhäuser Verlag, Basel, 1961 (Topological spaces, Academic Press, New York, 1965).

6. M. Mandelker, *Prime z-ideal structure of* C(**R**), Fund. Math. **63** (1968), 145–166.

7. M. Mandelker, *Prime ideal structure of rings of bounded continuous functions*, Proc. Amer. Math. Soc. **19** (1968), 1432–1438.

8. D. Plank, *On a class of subalgebras of $C(X)$ with applications to $\beta X — X$*, Fund. Math. (to appear).

9. S. M. Robinson, *The intersection of the free maximal ideals in a complete space*, Proc. Amer. Math. Soc. **17** (1966), 468–469.

10. W. J. Thron, *Topological structures*, Holt, Rinehart and Winston, New York, 1966.

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